N. Bohr, Phil. Mag., 25, 10 (1913).

II. *On the Theory of the Decrease of Velocity of Moving Electrified Particles on passing through Matter.* By N. Bohr, *Dr. phil. Copenhagen* *.

WHEN cathode-rays or α- and β-rays penetrate through matter their velocity decreases. A theory of this phenomena was first given by Sir J. J. Thomson †. In the calculation of this author the cathode- and β-rays are assumed to lose their velocity by collisions with the electrons contained in the atoms of the matter. The form of the law, found by this calculation, connecting the velocity of the particles and the thickness of matter traversed, has been recently shown by Whiddington ‡ to be in good agreement with experiments. Somewhat different conceptions are used in the calculation of Sir J. J. Thomson on the absorption of α-rays, as the latter, on account of their supposed greater dimensions, are assumed to lose their velocity by collisions, not with the single electrons but with the atoms of the matter considered as entities.

According to the theory given by Professor Rutherford § of the scattering of α-rays by matter, the atoms of the matter are supposed to consist of a cluster of electrons kept together by attractive forces from a nucleus. This nucleus, which possesses a positive charge equal to the sum of the negative charges on the electrons, is further supposed to be the seat of the essential part of the mass of the atom, and to have dimensions which are exceedingly small compared with the dimensions of the atom. According to this theory an α-particle consists simply of the nucleus of a helium atom. We see that after such a conception there is no reason to discriminate materially between the collisions of an atom with an α- or β-particle—apart of course from the differences due to the difference in their charge and mass.

An elaborate theory of the absorption and scattering of α-rays, based on Professor Rutherford's conception of the constitution of atoms, was recently published by C. G. Darwin ||. In the theory of this author the α-particles simply penetrate the atoms and act upon the single electrons contained in them, by forces varying inversely as the square

* Communicated by Prof. E. Rutherford, F.R.S.

† J. J. Thomson, 'Conduction of Electricity through Gases,' pp. 370–382.

‡ R. Whiddington, Proc. Roy. Soc. A. lxxxvi. p. 360 (1912).

§ E. Rutherford, Phil. Mag. xxi. p. 669 (1911).

|| C. G. Darwin, Phil. Mag. xxiii. p. 907 (1912).

of the distance apart*. By help of some further simple assumptions about the distribution of the electrons in the atoms and the effect of the forces acting upon them, Darwin obtained results for the scattering as well as for the absorption of the rays, which agree approximately with the experiments.

The above theories make use, however, of some special assumptions which seem to me to be open to objections of a principal character, and I have in this paper made an attempt to treat the problem in a somewhat different manner. The theory in question assumes that the loss of velocity of a moving electrified particle in passing through matter is due to a transfer of kinetic energy to the electrons of the atoms with which it collides. If we assume that the effect of the forces which keep the electrons in their position—or their orbits—inside the atoms can be neglected during the very short collisions between the electrons and the particles, we can very simply calculate the orbits of the electrons during the collisions, and consequently the energy transferred to them and the loss of velocity of the particle. If, however, we integrate the total loss of energy due to all the electrons in the matter, we get in this way an infinitely great value for the absorption. Sir J. J. Thomson, in his above mentioned theory of the decrease of velocity of cathode-rays, avoids this difficulty by introducing, as an effective limit for the action of the electrons on the velocity of the particles, a distance comparable in size with the distance apart of the single electrons in the atoms. This limit is chosen from the consideration that for distances greater than this, the effect of the different electrons on the moving particles will mutually disturb each other. The simultaneous influence of the different electrons on the particles will, as it will be seen, highly affect the deflexions of the particles for the distances in question, and the limit mentioned will therefore hold for the calculation of the scattering of the rays. The limit will, however, not hold for the calculation of the decrease of velocity of the particles; for, on account of the great velocity, the motion of the particles will be very slightly affected by collisions in which the distance of the electrons from the path of the particle is of the order of magnitude assumed for the distance apart of the electrons in the atoms. The forces exerted by a particle on an electron, and consequently the energy transferred to the

* Corresponding assumptions are also used by Sir J. J. Thomson in a recent paper on the ionization of moving electrified particles, Phil. Mag. xxiii. p. 449 (1912).

latter by the collision, will therefore be very nearly independent of the simultaneous effect of other electrons on the particle.

Darwin, in his theory of absorption of α-rays, proceeds in another way and avoids the difficulty by assuming that the forces on the electrons from the side of the atoms can be neglected during the very short and violent collisions between an electron and an α-particle, which occur when the particle traverses the same atom to which the electron belongs; and further, that the velocity of the α-particle will be unaltered if the particle during its path does not enter the atom. Using these assumptions and comparing the theory with the experiments, Darwin finds values for the diameter of the atoms which decrease for increasing atomic weight, and which for the lightest elements are several times greater than the generally adopted values for this quantity, and for the heaviest elements several times smaller. It seems, however, to me not to be justifiable to take the surface of the atoms as the limit of the effect of the electrons in the atoms on the particles. Outside an atom the forces on the particle from the electrons and the central positive charge will certainly very nearly neutralize each other; but the decrease of velocity of the particles depends only on the motion of the electrons during the collision, and not on the total force exerted on the particle by the whole atom, the latter force producing only the scattering of the rays.

We can, however, get a natural limit for the effect of the electrons on the velocity of the moving particles by taking into account the forces by which the electrons are kept in their positions in the atoms. Under the influence of these forces the electrons will have a sort of vibratory motion if they are disturbed by an impulse from outside. We see immediately that the forces in question will materially alter the motion of the electrons during the collision, and consequently the loss of energy of the particle, if the time of vibration of the electrons is of the same order of magnitude as the time of collision, *i. e.*, the time which the particle takes to travel through a distance of the same order of magnitude as the shortest distance apart of the electron from the path of the particle *. We see, further, that the effect of the electrons on the velocity of the particle will decrease very rapidly with the distance of the electrons from the particle, if this distance is so great that the time of collision is great compared with the time of vibration. The effective

* Compare J. J. Thomson, *loc. cit.* Phil. Mag. xxiii. p. 454 (1912).

limit for the action of the electrons on the velocity of the particles—and consequently the value of the absorption of the rays—which we get in this way will depend purely on the frequency of the electrons and the velocity of the particles, and may for the same velocity of the particles be very different for the different electrons inside the same atom, according to their different frequencies. The limit in question will, at least for some of the electrons in elements of high atomic weight, in which elements the existence of vibrations with very high frequencies is observed, be much smaller than for the electrons in the elements of low atomic weight. This circumstance seems, as it will be shown, to account for the comparatively much smaller absorption of such elements for the same weight of matter per $cm.^2$

It will be perceived that the theory of the decrease of velocity of moving electrified particles on passing through matter in this form bears a great analogy to the ordinary electromagnetic theory of dispersion ; the different times of vibration for the different wave-lengths considered in the theory of dispersion is here replaced by the different times of collision of particles of different velocities and at different distances from the electrons. In fact it will be shown, that the information about the number and the frequency of the electrons in the atoms, which we get from the theory of dispersion, will enable us to calculate values for the absorption of α-rays for the lightest elements which are in very close agreement with the observed values. Since, however, the decrease in the effect of the electrons corresponding to an increase in their frequency is much more rapid for the dispersion than for the loss of velocity of moving particles, it seems possible by considering the latter to get more information about the higher frequencies in the atoms, and from this some more information about the internal structure of the atoms.

In considering the collisions between the electrons and the particles we shall at first neglect the forces from the side of the atoms. Let E and M be the charge and the mass of a particle, and e and m be corresponding quantities for an electron. Let us further assume that the electron is at rest, and that the particle has a velocity V before the collision, and let the distance apart of the electron from the path of the particle before the collision be p ; then the calculation

of the orbits gives *

$$\sin^2 \vartheta = \frac{1}{1+\frac{p^2V^4}{e^2E^2}\left(\frac{mM}{M+m}\right)^2},$$

where 2ϑ is the angle through which the direction of the relative motion is deflected by the collision. For the sake of brevity we shall in the following use the notation

$$\lambda = \frac{eE(M+m)}{V^2mM}.$$

The velocity of the electron after the collision will make an angle equal to $\frac{\pi}{2}-\vartheta$ with the path of the particle before the collision, and its value will be given by

$$v = V\frac{M}{M+m}2\sin\vartheta.$$

The energy transferred to the electron by the collision is consequently equal to

$$Q_0 = \frac{2mM^2V^2}{(m+M)^2}\sin^2\vartheta. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

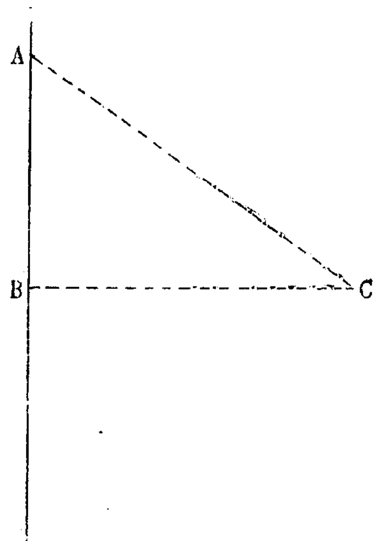
Further, we easily find that the displacement of the electron in a direction perpendicular to the path of the particle, at the moment in which the electrons and the particle are nearest each other, is equal to $\frac{eE}{mV^2}\cos\vartheta$. We see that ϑ will be very small, and the velocity of the electron after the collision very nearly perpendicular to the path of the particle if p is great in comparison with λ; in this case the displacement of the electron during the collision will further be very small in comparison with p.

Now proceeding to consider the effect of the forces on the electrons from the side of the atoms, we shall for the present assume that the frequency of the electrons is so small that the time of vibration is very long in comparison with the time of collision, for collisions in which p is of the same order of magnitude as λ; this will, in fact, be satisfied for the lightest elements, as we shall see later. In this case we shall consequently only have to consider the influence

* Compare J. J. Thomson, 'Conduction of Electricity through Gases,' p. 376, and Phil. Mag. xxiii. p. 449 (1912); C. G. Darwin, *loc. cit.* p. 903.

of the forces in question for collisions in which p is great in comparison with λ. This simplifies the calculation very much, because then we can assume that the displacement during the collision is negligibly small in proportion to p. In the following calculation we shall consider separately the motion of the electrons perpendicular and parallel to the path of the particle; the total energy transferred to the electrons during the collisions will be the sum of the energy corresponding to these two motions.

In the figure the line AB represents the path of the particle, which in the collisions considered here (*i. e.*, p great



in proportion to λ) will be very nearly a straight line. Further, A is the position of the particle at the time t, and C is the mean position of the electron. BC is perpendicular to AB. According to the above notation, $\overline{BC}=p$; and assuming that the particle will be at B when the time is 0, we have $\overline{AB}=V.t$.

For the force acting on the electron in the direction CB, we now get

$$F_1=eE\frac{\overline{BC}}{\overline{AC}^3}=\frac{eEp}{(V^2t^2+p^2)^{\frac{3}{2}}}=m.\phi(t).$$

For the equation of motion of the electron perpendicular to the path of the particle we get

$$\frac{d^2x}{dt^2}+n^2x=\phi(t),$$

in which n is the frequency corresponding to the forces in question.

The solution of this equation subject to the condition that

$$x=0 \quad \text{and} \quad \frac{dx}{dt}=0 \quad \text{for} \quad t=-\infty, \text{ is*}$$

$$x=\frac{1}{n}\int_{-\infty}^{t}\sin n(t-z)\,.\,\phi(z)dz\,;\quad \frac{dx}{dt}=\int_{-\infty}^{t}\cos n(t-z)\,.\,\phi(z)dz.$$

We have in the above expression assumed that the electron was at rest before the collision with the particle; if we assume that the electrons are in motion in the atoms before the collision (the dimension of their orbits must, however, for the justification of the above calculations be small in proportion to p; for the fulfilment of this condition see later on p. 20), the effect will only be an introduction of some terms in the expressions for x and $\frac{dx}{dt}$ which will again disappear in the expression for the mean value of the energy transferred.

For the sum of the kinetic energy of the electron at the time t, and its potential energy due to its displacement relative to the rest of the atom, we have now

$$\frac{m}{2}\left(\frac{dx}{dt}\right)^2+\frac{mn^2}{2}x^2=\frac{m}{2}\left[\int_{-\infty}^{t}\cos nz\,.\,\phi(z)dz\right]^2$$
$$+\frac{m}{2}\left[\int_{-\infty}^{t}\sin nz\,.\,\phi(z)dz\right]^2.$$

For the energy transferred to the electron by the collision, due to motion perpendicular to the path of the particle, we now get, observing that in this case $\phi(z)$ is an even function of z,

$$Q_1=\frac{m}{2}\left[\int_{-\infty}^{+\infty}\cos nz\,.\,\phi(z)dz\right]^2,$$

and introducing for $\phi(z)$

$$Q_1=\frac{1}{2}\frac{e^2}{m}E^2p^2\left[\int_{-\infty}^{+\infty}\frac{\cos nz}{(V^2z^2+p^2)^{\frac{3}{2}}}dz\right]^2,$$

$$Q_1=\frac{2e^2E^2}{mV^2p^2}\cdot f^2\left(\frac{np}{V}\right),$$

* See Lord Rayleigh, 'Theory of Sound,' i. p. 75. For the following analysis compare also J. H. Jeans, 'Kinetic Theory of Gases,' p. 198.

where
$$f(x)=\frac{1}{2}\int_{-\infty}^{+\infty}\frac{\cos xz}{(z^2+1)^{\frac{3}{2}}}dz$$

is represented for all values of x by the convergent series

$$f(x)=1-\frac{1}{1.1.2}\frac{3}{1.2}\left(\frac{x}{2}\right)^4-\frac{1}{1.2.1.2.3}\left(\frac{3}{1.2}+\frac{5}{2.3}\right)\left(\frac{x}{2}\right)^6\cdots$$
$$-\frac{1}{(n-1)!\,n!}\left(\frac{3}{1.2}+\frac{5}{2.3}+\cdots\frac{2n-1}{(n-)n}\right)\left(\frac{x}{2}\right)^{2n}\cdots$$
$$+\left(2\gamma+2\log\frac{x}{2}-1\right)\left(\left(\frac{x}{2}\right)^2+\frac{1}{1.1.2}\left(\frac{x}{2}\right)^4+\frac{1}{1.2.1.2.3}\left(\frac{x}{2}\right)^6\cdots\right.$$
$$\left.+\frac{1}{(n-1)!\,n!}\left(\frac{x}{2}\right)^{2n}\cdots\right),$$

where γ is Euler's constant, $\gamma=0{\cdot}5772$.

When x is large, $f(x)$ is represented by the asymptotic series

$$f(x)\sim\sqrt{\frac{\pi}{2}}\,.\,e^{-x}\sqrt{x}\left(1+\frac{1.3}{8x}-\frac{1.3.5}{1.2}\left(\frac{1}{8x}\right)^2+\frac{1.3.1.3.5}{1.2.3}\left(\frac{1}{8x}\right)^3-\cdots\right.$$
$$\left.(-1)^{n+1}\frac{1.3.5\ldots(2n-3).1.3\ldots(2n-1)}{n!\,(8x)^n}\cdots\right).$$

For the force acting on the electrons in the direction parallel to the path of the particles we have now (see figure, p. 15)

$$F_2=eE\frac{\overline{AB}}{\overline{AC}^3}=\frac{eEVt}{(V^2t^2+p^2)^{\frac{3}{2}}}=m\,.\,\psi(t).$$

For the energy transferred to the electron during the collision we get in the same manner as above ($\psi(t)$ an uneven function of t)

$$Q_2=\frac{m}{2}\left[\int_{-\infty}^{+\infty}\sin nz\,.\,\psi(z)dz\right]^2,$$

and introducing for $\psi(z)$

$$Q_2=\frac{1}{2}\frac{e^2}{m}E^2V^2\left[\int_{-\infty}^{+\infty}\frac{z\sin nz}{(V^2z^2+p^2)^{\frac{3}{2}}}dz\right]^2,$$

$$Q_2=\frac{2e^2E^2}{mV^2p^2}\,.\,g^2\left(\frac{np}{V}\right),$$

where

$$g(x) = -\frac{1}{2}\int_{-\infty}^{+\infty} \frac{z \sin xz}{(z^2+1)^{\frac{3}{2}}} dz = \frac{x}{2}\int_{-\infty}^{+\infty} \frac{\cos xz}{(z^2+1)^{\frac{1}{2}}} dz = f'(x),$$

in which $f(x)$ is the same function as above.

While for the motion perpendicular to the direction of the particle the energy transferred is always smaller than the energy calculated by considering the electrons as free, this is not the case for the motion parallel to the path of the particle.

For the total energy transferred to the electron by the collision we have now

$$Q = Q_1 + Q_2 = \frac{2e^2 E^2}{mV^2 p^2} \cdot P\left(\frac{np}{V}\right), \quad . \quad . \quad . \quad (2)$$

where $P(x) = f^2(x) + g^2(x)$ is equal to 1 for $x = 0$, and decreases very rapidly for increasing values of x, when x is great; for $x = 0$ we notice that $P'(x) = 0$.

Let us now consider a particle passing through matter. Let us assume that the numbers of atoms per unit volume is N, and that each atom contains r electrons of frequency n. Let further a be a constant, great in comparison with λ, but small in comparison with V/n (see p. 15), we then get for the total energy dT transferred to the electrons when the particle travels through a distance dx

$$dT = Nr\left[\int_0^a Q_0 2\pi p dp + \int_a^\infty Q 2\pi p dp\right] dx;$$

by help of (1) and (2) we get

$$dT = \frac{4\pi e^2 E^2 N r}{mV^2}\left[\int_0^a \frac{pdp}{p^2+\lambda^2} + \int_a^\infty \frac{1}{p} P\left(\frac{np}{V}\right) dp\right] dx.$$

Neglecting $(\lambda/a)^2$ (see above), we get

$$dT = \frac{4\pi e^2 E^2 N r}{mV^2}\left[\log\left(\frac{a}{\lambda}\right) + \int_{\frac{an}{V}}^{\infty} \frac{1}{z} P(z) dz\right] dx,$$

$$dT = \frac{4\pi e^2 E^2 N r}{mV^2}\left[\log\left(\frac{a}{\lambda}\right) - \log\left(\frac{an}{V}\right) \cdot P\left(\frac{an}{V}\right) - \int_{\frac{an}{V}}^{\infty} \log z \, . \, P'(z) dz\right] dx.$$

According to our assumption, $\frac{an}{V}$ is very small, and we can

therefore put $P\left(\frac{an}{V}\right)=1$, and further take the limit for the integral to be 0 and ∞, $(P'(0)=0)$.

Putting

$$\int_0^\infty \log z P'(z) dz = -\log k,$$

we thus get

$$dT = \frac{4\pi e^2 E^2 N r}{m V^2} \log\left(\frac{V^3 k M m}{neE(M+m)}\right) dx.$$

I have calculated k by help of the above formulæ for $f(x)$ and found

$$k = 1{\cdot}123.$$

If we assume that the atoms contain electrons corresponding to different frequencies, and if we denote the frequencies of the r electrons in each atom by $n_1, n_2, \ldots n_r$, we get

$$dT = \frac{4\pi e^2 E^2 N}{m V^2} dx \sum_{s=1}^{s=r} \log\left(\frac{V^3 k M m}{n_s eE(M+m)}\right). \quad . \quad . \quad (3)$$

Since dT is equal to the decrease in the kinetic energy of the particle, *i. e.*, in $\frac{1}{2}MV^2$, we have

$$\frac{dV}{dx} = -\frac{4\pi e^2 E^2 N}{m M V^3} \sum_{s=1}^{s=r} \log\left(\frac{V^3 k M m}{n_s eE(M+m)}\right). \quad . \quad (4)$$

In establishing the formula (4) we have only considered the interaction between the particle and the electrons, and not the interaction between the particle and the central charge in the atoms; as Darwin * has shown, the effect of the latter interaction will, however, be negligibly small in comparison with the former: this conclusion will hold unaltered for the theory in the form it is given here.

The formula (4) expresses the rate of decrease of velocity of moving electrified particles as a function of the velocity of the particles and the number and frequencies of the electrons.

If V is very great we can neglect the variation in the logarithmic term, and get for the relation between V and the distance the particles have travelled through matter, denoting the velocity for $x=0$ by V_0,

$$V_0^4 - V_x^4 = ax, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where

$$a = \frac{16\pi e^2 E^2 N}{m M} \sum_{s=1}^{s=r} \log\left(\frac{V_0^3 k M m}{n_s eE(M+m)}\right).$$

* Darwin, *loc. cit.* p. 905.

This relation is of the same form as the one deduced by Sir J. J. Thomson, and shown by Whiddington to hold approximately for cathode-rays (see p. 10). For still greater velocities, corresponding to the fastest β-rays, the form of the relation between V and x will be altered on account of the rapid increase in the mass of the particles if their velocity is very near to the velocity of light (see later, p. 29).

For smaller values of the velocities of the particles the logarithmic term will be of material influence on the relation between V and x, the effect being of the sense as to diminish the power of V on the right side of the equation (5). This is in agreement with experiments on α-rays.

If we assume that the number of electrons in an atom is proportional to the atomic weight, and if we consider that the atoms of elements of increasing atomic weight contain electrons of increasing frequency, we see immediately that the formula (4) accounts for some of the principal features of the absorption of α-rays by different elements. It accounts for the fact that the absorption for equal weight of matter per square centimetre decreases for elements of increasing atomic weight *. It accounts further for the fact that the relative absorption for different elements varies with the velocity of the α-rays, the absorption for the heavier elements being comparatively greater for greater velocities of the rays†.

For a closer numerical comparison between the theory and the experiments we must, however, observe that in the deduction of the formula (4) we have made use of some assumptions about the magnitude of the frequency and velocity of the electrons which may not be satisfied with all the electrons in the atoms considered, for the velocities of the particles in question. These assumptions are :—

(1) That the frequency n is small compared with V/λ;

(2) That the velocity τ of the electrons in their undisturbed orbits is small compared with the velocity of the particles;

(3) That the linear dimensions ρ of the orbits in question are small compared with $\frac{V}{n}$ (see p. 16).

As regarding the order of magnitude τ and ρ are connected by the relation $\tau = n\rho$, we see that condition (2) is fulfilled at the same time as (3). The calculations involves still the assumption,

(4) that the displacement of the electrons caused by the

* W. H. Bragg and R. Kleeman, Phil. Mag. x. p. 318 (1905).

† T. S. Taylor, Phil. Mag. xviii. p. 604 (1909).

forces from the particle is small compared with the dimensions of their undisturbed orbits, for such collisions for which we have to take the forces from the side of the atom into account; or in other words, that nothing of the same sort as ionization will occur for such collisions. As, however, the forces between the particles and the electrons for the same distance are of the same order of magnitude as the forces which act upon the electron from the central charge and the other electrons, we see that condition (4) is satisfied if condition (3) is.

It seems very difficult to account accurately for the alterations in the result, if the above assumptions are not satisfied; but it is easy to see that if the ratios $\frac{n\lambda}{V}$ and $\frac{n\rho}{V}$ are small quantities, then the corrections in the result will be proportional to the squares of these ratios*.

Comparison with Experiments.

I. *α-rays.*

We shall at first consider the absorption of α-rays, as the behaviour of these rays, on account of the small scattering, is much more accurately known than the corresponding facts for β- or cathode-rays.

Absolute measurements of the variation of the velocity of α-rays with the thickness of matter traversed have lately been made by Geiger † for air. This author found that the relation

$$V^3 = KR, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where V is the velocity of the α-rays and R the corresponding range of the rays in air, was satisfied with great accuracy for a very great part of the path of the rays. In determining K, we have that the range in air of α-rays from radium C is 7·06 cm. (at 76 cm. and 20° C.) ‡, and that the initial velocity of these rays is § $1{\cdot}98 \,.\, 10^9$ cm./sec.; this gives $K = 1{\cdot}10 \,.\, 10^{27}$.

Elaborate measurements of the relative absorption-coefficients of different elements for α-rays corresponding to different ranges in air have been made by Taylor ‖. The

* Compare Darwin, *loc. cit.* p. 902.
† H. Geiger, Proc. Roy. Soc. A. lxxxiii. p. 505 (1910)
‡ Bragg and Kleeman, *loc. cit.* p. 318.
§ E. Rutherford, Phil. Mag. xii. p. 358 (1906). (The above value for V is Rutherford's value for V . M/E, multiplied by $4{\cdot}87 \,.\, 10^3$, *i. e.* the value of E/M for helium.)
‖ T. S. Taylor, Phil. Mag. xviii. p. 604 (1909).

ranges in air of the α-rays, when entering the absorbing sheets, varied in these experiments from about 5 to about 2 cm. The figures in the table below for the absorption relative to air are obtained by interpolation in Taylor's Tables II. and III. (*loc. cit.* pp. 608–610), the range quoted is the mean value of the ranges for the α-rays entering and leaving the absorption sheets, and the values for the absorption are the mean values calculated from the different series of experiments using same absorbing material.

Range in air	2·24.	4·87.
Hydrogen	0·267	0·224
Air	1·00	1·00
Aluminium..................	$1{\cdot}69 \,.\, 10^3$	$1{\cdot}75 \,.\, 10^3$
Tin	$2{\cdot}33 \,.\, 10^3$	$2{\cdot}56 \,.\, 10^3$
Gold.........................	$4{\cdot}71 \,.\, 10^3$	$5{\cdot}57 \,.\, 10^3$
Lead.........................	$3{\cdot}06 \,.\, 10^3$	$3{\cdot}53 \,.\, 10^3$

The ranges 2·24 and 4·87 are chosen as those corresponding to the velocities $1{\cdot}35 \,.\, 10^9$ and $1{\cdot}75 \,.\, 10^9$ according to the formula (5). According to the same formulæ we further get that for these velocities, $\frac{dV}{dx}$ in air is equal to respectively $-2{\cdot}01 \,.\, 10^8$ and $-1{\cdot}20 \,.\, 10^8$. From this we get by help of the above table the following values for $-\frac{dV}{dx}$:

Velocity	$1{\cdot}35 \,.\, 10^9$.	$1{\cdot}75 \,.\, 10^9$
Hydrogen	$5{\cdot}4 \,.\, 10^7$	$2{\cdot}7 \,.\, 10^7$
Air	$2{\cdot}01 \,.\, 10^8$	$1{\cdot}20 \,.\, 10^8$
Aluminium..................	$3{\cdot}4 \,.\, 10^{11}$	$2{\cdot}1 \,.\, 10^{11}$
Tin	$4{\cdot}7 \,.\, 10^{11}$	$3{\cdot}1 \,.\, 10^{11}$
Gold.........................	$9{\cdot}5 \,.\, 10^{11}$	$6{\cdot}7 \,.\, 10^{11}$
Lead.........................	$6{\cdot}1 \,.\, 10^{11}$	$4{\cdot}2 \,.\, 10^{11}$

Hydrogen.

Comparing the above values with the theory, we shall start with hydrogen as the substance for which the assumptions mentioned on p. 20 are satisfied in the highest degree.

From the formula (4), p. 19, we get, putting

$e=4{\cdot}65.10^{-10}$, $E=2e$, $e/m=5{\cdot}31.10^{17}$, $E/M=1{\cdot}46.10^{14}$,

and $N=2{\cdot}59.10^{19}$ (at 76 cm. and 20° C.),

for

$$\left.\begin{aligned} &V=1{\cdot}35\,.\,10^{9},\quad \frac{dV}{dx}=4{\cdot}42\,.\,10^{6}\sum_{s=1}^{s=r}\left(\log\left(n_s\,.\,10^{-19}\right)+0{\cdot}59\right)\\ &\text{and for}\\ &V=1{\cdot}75\,.\,10^{9},\quad \frac{dV}{dx}=2{\cdot}03\,.\,10^{6}\sum_{s=1}^{s=r}\left(\log\left(n_s\,.\,10^{-19}\right)-0{\cdot}18\right) \end{aligned}\right\} \quad (7)$$

From experiments on the refraction and dispersion in hydrogen, and the discussion of these experiments according to Drude's theory, C. and M. Cuthbertson find that a hydrogen molecule in its normal state contains 2 electrons of frequency $n=2{\cdot}21\,.\,10^{16}$ *.

Putting $r=2$ and $n_1=n_2=2{\cdot}21.10^{16}$ in the above formulæ, we get

$$\text{for}\quad V=1{\cdot}35\,.\,10^{9},\quad \frac{dV}{dx}=-4{\cdot}9\,.\,10^{7},$$

and

$$\text{for}\quad V=1{\cdot}75\,.\,10^{9},\quad \frac{dV}{dx}=-2{\cdot}6\,.\,10^{7}.$$

These values are in close agreement with the values for $\frac{dV}{dx}$ in the table on p. 22, *i. e.* respectively

$$\frac{dV}{dx}=-5{\cdot}4\,.\,10^{7}\quad\text{and}\quad\frac{dV}{dx}=-2{\cdot}7.\,10^{7}.$$

The small differences between the calculated and the observed values are not greater than was to be expected, as the values calculated, on account of possible experimental errors in the entering constants, are not certain within more than about 10 p.c. We shall further here examine to what

* C. and M. Cuthbertson, Proc. Roy. Soc. A. lxxxiii. p. 166 (1909); see also Drude, *Ann. d. Phys.* xiv. p. 714 (1904). (The agreement with Drude's theory is, however, not quite satisfactory, as the number of electrons works out somewhat less than 2. A probable explanation of this fact seems to be that the frequency of the electrons is not the same for displacements in all directions, a circumstance not to be expected in a system which has at most one axis of symmetry, *i. e.* the axis of the diatomic molecule. This question will be discussed in a later paper; for the present we shall use the above value for n, as the influence on the result of the correction in this value, which will follow from the discussion referred to, will not be greater than the inevitable errors due to the uncertainty in the other experimental constants entering into the calculations.)

degree the conditions on p. 20 are satisfied in the case in question. For $V=1{\cdot}75\,.\,10^9$, we get

$$\lambda=\frac{eE(M+m)}{V^2Mm}=1{\cdot}6\,.\,10^{-10} \quad \text{and} \quad V/n=0{\cdot}8\,.\,10^{-7}.$$

We see that the first condition is amply satisfied. We have, further, that the maximum value to be assumed for the quantity ρ on p. 20 is about 10^{-8}, *i. e.* the "radius" of a hydrogen molecule; we thus get the maximum value to be assumed for $\frac{\rho n}{V}$ equal to about 0·1. As the corrections due to finite values for $\frac{\rho n}{V}$, as mentioned on p. 20, are proportional to the square of this quantity, we must, therefore, expect them very small in the considered case.

It may here be remarked that the above value for V/n shows that the effective limit, mentioned on p. 12, for the action of the electrons on the velocity of the particles, for hydrogen and for particles of the velocity considered is about 8 times the radius of the molecules; for β-rays of velocity near the velocity of light the limit in question would be more than 100 times the radius of the molecules.

We see that the absorption of α-rays in hydrogen can be satisfactorily accounted for by assuming the same number of electrons per molecule and the same frequencies, as those assumed in order to explain the refraction and dispersion in this gas. It may here be mentioned, that if we assumed that the hydrogen molecule contained more than two electrons, the theory shows that the frequencies of the other electrons must be extremely high, as the absorption due to them, after the above calculation, cannot amount to more than about 10 per cent. of the absorption due to the two electrons considered. Thus assuming that the molecule contains further two electrons, we get that n for them must be at least of the order of magnitude of 10^{18}; a value for n which seems difficult to reconcile with experiments on characteristic Röntgen rays (comp. later, p. 26). If we adopt Rutherford's conception of the constitution of atoms, we see that the experiments on absorption of α-rays very strongly suggest, that a hydrogen atom contains only one electron outside the positively charged nucleus.

Helium.

For helium there is no measurement of the absorption coefficient for different velocities; the only experiment with this gas is a determination by Adams * of the range in

* E. P. Adams, Physical Review, xxiv. p. 113 (1907).

helium of α-rays from polonium. Adams finds the absorption in helium a little bigger than in hydrogen; the ratio being 1·15.

According to Cuthbertson's experiments* and Drude's theory the dispersion in helium can be explained by assuming two electrons per atom (the calculated value is 2·3) of a frequency $n=3 \cdot 72 \, . \, 10^{16}$.

Introducing these values for r and n in the formula (4), p. 19, we get values for $\frac{dV}{dx}$ in helium which are a little smaller than those found above for hydrogen, the ratio being 0·92 for $V=1 \cdot 75 \, . \, 10^9$ and 0·90 for $V=1 \cdot 35 \, . \, 10^9$.

If this disagreement is real (Adams states (*loc. cit.* p. 111) that the purity of the gases used was not secured very effectively; a little contamination of the helium with heavier gases will explain the disagreement in question) it suggests that even for helium the neglected corrections will be of sensible influence. As, however, the quantity $\frac{\rho n}{V}$ is to be assumed about twice as large for helium as for hydrogen, we see, on the other hand, that the corrections, being about 30 per cent. for helium, will not be more than about 10 per cent. for hydrogen. The following results for oxygen and aluminium seem, however, to indicate that the corrections considered are much smaller.

The value $r=2$ for the number of electrons in a helium atom, indicated by experiments on dispersion and on absorption of α-rays, is what we, adopting Rutherford's theory of atoms, necessarily must conclude from the behaviour of α-rays, according to which helium atoms formed from α-particles will only contain 2 electrons outside the central nucleus.

Oxygen.

For the ratio between the absorption in oxygen and in air, Adams (*loc. cit.* p. 113) found 1·03, and according to the table, p. 22, we therefore get for oxygen

$$\text{for} \quad V=1 \cdot 35 \, . \, 10^9, \quad \frac{dV}{dx}=-2 \cdot 07 \, . \, 10^8,$$

and

$$\text{for} \quad V=1 \cdot 75 \, . \, 10^9, \quad \frac{dV}{dx}=-1 \cdot 24 \, . \, 10^8.$$

* C. & M. Cuthbertson, Proc. Roy. Soc. A. lxxxiv. p. 13 (1910).

By comparison with the formulæ (7), p. 23, we now get

$$\sum_{s=1}^{s=r}(\log(n_s \, . \, 10^{-19})+0{\cdot}59)=-47,$$

$$\sum_{s=1}^{s=r}(\log(n_s \, . \, 10^{-19})-0{\cdot}18)=-61.$$

From this we get at first by subtraction

$$r \, . \, 0{\cdot}77=14 \quad \text{or} \quad r=18.$$

According to Rutherford's theory of atoms we should expect 16 electrons in an oxygen molecule. The agreement between this value and the above value for r is very satisfactory.

From the above we get further

$$\sum_{s=1}^{s=r} \log(n_s \, . \, 10^{-19})=-58.$$

From experiments on dispersion * we have that an oxygen molecule contains 4 electrons of frequency $2{\cdot}25 \, . \, 10^{16}$; we get, therefore,

$$\sum_{s=5}^{s=r} \log(n_s \, . \, 10^{-19})=-58+4 \, . \, 6{\cdot}1=-34.$$

If we for the present assume that the other 12 electrons supposed contained in an oxygen molecule have equal frequencies n', we get

$$\log(n' . \, 10^{-19})=-2{\cdot}8, \quad \text{and} \quad n'=0{\cdot}6 \, . \, 10^{18}.$$

We know very little about the higher frequencies in oxygen, but we can get some estimation of what we should expect, from experiments on characteristic Röntgen rays. Whiddington † has found that the velocity of an electron just sufficient to excite the characteristic Röntgen rays in an element is equal to $A \, . \, 10^8$ cm./sec., where A is the atomic weight of the element in question. The energy possessed by such an electron is $\frac{m}{2} A^2 . \, 10^{16}$. According to Planck's theory of radiation we further have that the smallest quantity of energy which can be radiated out from an atomic vibrator is equal to $v \, . \, k$, where v is the number of vibrations per second and $k=6{\cdot}55 \, . \, 10^{-27}$. This quantity must be expected to be equal

* C. & M. Cuthbertson, *loc. cit.* p. 166.

† R. Whiddington, Proc. Roy. Soc. A. lxxxv. p. 323 (1911).

to, or at least of the same order of magnitude as, the kinetic energy of an electron of velocity just sufficient to excite the radiation: putting them equal we get $vk = \frac{m}{2} A^2 . 10^{16}$, and from this $v = A^2 . 6{\cdot}7 . 10^{14}$. No experiments on characteristic Röntgen rays are made for oxygen, but if we assume Whiddington's law to hold for this element, and put $A = 16$ in the above expression for v, we get $v = 1{\cdot}7 . 10^{17}$, and accordingly for the frequency $n = 2\pi v = 1{\cdot}1 . 10^{18}$. The agreement as to the order of magnitude between this value and the above value for n', calculated from the absorption of α-rays, is remarkable.

An estimation of the magnitude of the corrections to be introduced in the formula (4) in case of oxygen, involves a discussion about the relation between the frequencies and the dimensions of the orbits of the electrons in the interior of the atoms; and must therefore be postponed till the later paper referred to in the note on p. 23.

For *aluminium*, *tin*, *gold*, and *lead*, we get in the same manner as for oxygen, by comparing the values in the table on p. 22 with the formula (4) on p. 19, the following values for r and $\Sigma \log (n_s . 10^{-19})$.

Substance.	r.	$-\sum_{s=1}^{s=r} (\log n_s . 10^{-19})$.	Atomic weight.
Aluminium	14	41	27
Tin................	38	94	119
Gold	61	126	197
Lead	65	132	207

According to Rutherford's theory we shall expect values for r equal to about the half of the atomic weight; we see that this is the case for aluminium, but that the values for r for the elements of higher atomic weight are considerably lower. The values found for $\Sigma \log n_s$ are of a magnitude to be expected if the atoms contained electrons of different frequencies varying from the order of magnitude of the frequencies observed by the dispersion in the transparent bodies to that of the characteristic Röntgen rays. It must, however, here be remarked that the magnitude of the corrections to be introduced in the formulæ (4) must be expected to increase with increasing atomic weight of the substance

considered, and that for the elements of higher atomic weight especially the values calculated for r are uncertain, as these values are determined by considering the difference in the absorption of α-rays of different velocities, and for these velocities the differences in the neglected corrections may be considerable.

II. *Cathode rays and β-rays.*

The most detailed measurements of the decrease of velocity of cathode rays in passing through matter have been made by Whiddington*. This author found, using cathode rays of velocity between about 5.10^9 and 9.10^9, that the variation in the velocity of the rays and the thickness of matter traversed was connected by a relation of the form given by the equation (5) on p. 19. The determination of the entering constant a gave

for Aluminium, $a=7{\cdot}32\,.\,10^{42}$; for Gold, $a=2{\cdot}54\,.\,10^{43}$:

and for Air at 760 mm. pressure of mercury and 15° C., $a=2{\cdot}0\,.\,10^{40}$.

From the expression for a on p. 19, we get, putting $v=7.10^9$ and introducing the values for r and $\Sigma \log n_s$ found above in considering the absorption of α-rays,

for Aluminium, $a=1{\cdot}9\,.\,10^{43}$; for Gold, $a=7{\cdot}3\,.\,10^{43}$;
and for Air, $a=1{\cdot}1\,.\,10^{40}$.

We see that the observed and calculated values agree as to the order of magnitude, but that the differences are very considerable, the values calculated for aluminium and gold being about three times greater than the values observed, and the value of air about half. It seems difficult to account for this disagreement, if it cannot in one way or other be referred to the extraordinary difficult experimental conditions. It may thus be remarked, that the ratio between the rates of decrease of velocity in aluminium and in air is found about five times smaller in Whiddington's experiments than in experiments with α-rays; a circumstance which seems difficult to reconcile with facts found as well by experiments with α-rays of different velocities as by comparing the results of experiments on β-rays with those for α-rays, *i. e.*, that the rate of decrease of velocity in different substances, calculated per number of atoms, is greater for substances of higher

* R. Whiddington, Proc. Roy. Soc. A, lxxxvi. p. 360 (1912).

atomic weight, and that the ratio between the rates of decrease increases with increasing velocity of the rays.

Measurements of the decrease of velocity of β-rays is made for very hard β-rays by W. Wilson*, and recently for slower rays by O. v. Baeyer†. The last author found by experiments with aluminium, and using β-rays, the velocity of which was between 1.10^{10} and 2.10^{10} cm./sec., that the variation in the velocity approximately satisfied a relation of the same form as that found by Whiddington. For a velocity of $1{\cdot}5.10^{10}$ he found the constant a equal to about $1{\cdot}1.10^{42}$.

From the expression for a on p. 19, we get for the velocity considered, introducing the values for r and $\Sigma \log n_s$, found for α-rays, and putting $M=1{\cdot}54\,m$, *i. e.* the longitudinal mass of an electron moving with a velocity equal to half the velocity of light (the influence of the alteration in the mass of the particles on the constant a is for this velocity already considerable, but the variation in the mass with the velocity is still too low to alter materially the form of the relation connecting V and x),

$$a=1{\cdot}7\,.\,10^{42}.$$

We see that the agreement for these faster rays is better than the one found above for cathode rays.

O. v. Baeyer has also made a few measurements of the decrease of velocity of β-rays in tin, copper, and platinum. The result of these experiments was that the rate of decrease for the same velocity varied approximately proportional to the density of the matter traversed; the elements of higher atomic weight seemed, however, to absorb a little less per same weight per cm.^{2}. These results are in conformity with what we should expect according to the theory.

Wilson found that the results of his experiments on the decrease of velocity of very hard β-rays in aluminium was in better conformity with an equation of the form $E_s-E_x=kx$, where E is the energy of the β-particle, than with the equation (4). This is, however, just what was to be expected according to the theory. For, on account of the very rapid increase of the β-particle with its velocity, when near to the velocity of light, the variation in V^2 is for such velocities small compared with the variation in the energy of the particle. By considering the equation (3) on p. 19, we consequently get that, the relation between the energy of the particle and the thickness of matter traversed for the velocities

* W. Wilson, Proc. Roy. Soc. A. lxxxiv. p. 141 (1910).
† O. v. Baeyer, *Physikalische Zeitschrift*, xiii. p. 485 (1912).

in question takes the same form as the one found by Wilson. From Wilson's Table II. (*loc. cit.* p. 147) we get

$$\text{for}\quad V=2{\cdot}8\,.\,10^{10},\quad \frac{dE_x}{dx}=-8{\cdot}0\,.\,10^{-6}.$$

From the equation (3) on p. 19, we get for this velocity, and using the same values for r and $\Sigma \log n_s$ as above,

$$\frac{dE_x}{dx}=-8{\cdot}8\,.\,10^{-6},$$

a value which is in satisfactory agreement with Wilson's value. The better agreement between theory and experiments for fast β-rays than for slower ones and for cathode rays is probably connected with the simpler experimental conditions for the fast rays, for the latter keep their original uniformity in velocity to a much higher degree in passing through matter than is the case for the slower rays.

Conclusions.

In this paper the theory of the decrease of velocity of moving electrified particles in passing through matter is given in a form, such that the rate of the decrease in the velocity depends on the frequency of vibration of the electrons in the atoms of the absorbing material.

It is shown that the absorption of α-rays in the lightest elements can be calculated from the information about the number and frequencies of the electrons in the atoms which we get from the theory of dispersion, and that the values are in good agreement with experiment. For elements of higher atomic weight, it is shown that the number and frequencies of the electrons which we must assume, according to the theory, in order to explain the absorption of α-rays are of the order of magnitude to be expected.

It is further shown that the theory can account for the form of the relations between the velocity of the rays and the thickness of matter traversed, found by experiments with cathode- and β-rays. The absolute agreement as to the magnitude of the constants entering in the relations in question, is very good for the fastest β-rays, but not so good for slower β-rays and for cathode rays; a fact which may be due to the very difficult experimental conditions for these latter rays.

Adopting Prof. Rutherford's theory of the constitution of atoms, it seems that it can be concluded with great certainty,

from the absorption of α-rays, that a hydrogen atom contains only 1 electron outside the positively charged nucleus, and that a helium atom only contains 2 electrons outside the nucleus ; the latter was necessarily to be expected from Rutherford's theory.

These questions and some further information about the constitution of atoms which may be got from experiments on the absorption of α-rays, will be discussed in more detail in a later paper.

I wish to express my sincere thanks to Prof. Rutherford for the kind interest he has taken in this work, and for the helpful advice he has given me.

Physical Laboratories,
The University, Manchester,
August 1912.

III. *On the Application of the Theory of Chemical Potential to the Thermodynamical Theory of Solutions. By* S. A. SHORTER, *B.Sc., Assistant Lecturer in Physics in the University of Leeds.*—Part III. *The Action of Gravity on a Solution. The Solute Potential. Extension of the Theory*.*

IN the parts of the present communication dealing with the action of gravity on a solution, and with the solute potential, the following new symbols will be used :—

G, the potential of gravity (defined so as to increase downwards) ;

$\psi_1(p, \theta)$, the chemical potential of the solid solute ;

$$S_0(s, p, \theta) \equiv \frac{\partial}{\partial s} f_0(s, p, \theta) ;$$

$$S_1(s, p, \theta) \equiv \frac{\partial}{\partial s} f_1(s, p, \theta) ;$$

$$P_1(s, p, \theta) \equiv \frac{\partial}{\partial p} f_1(s, p, \theta) ;$$

$$\Delta_1(s, p, \theta) \equiv \psi_1(p, \theta) - f_1(s, p, \theta) ;$$

$l_1(s, p, \theta)$ the heat of solution.

The new symbols used in the last section will be defined as they occur in the text.

* Communicated by the Author.